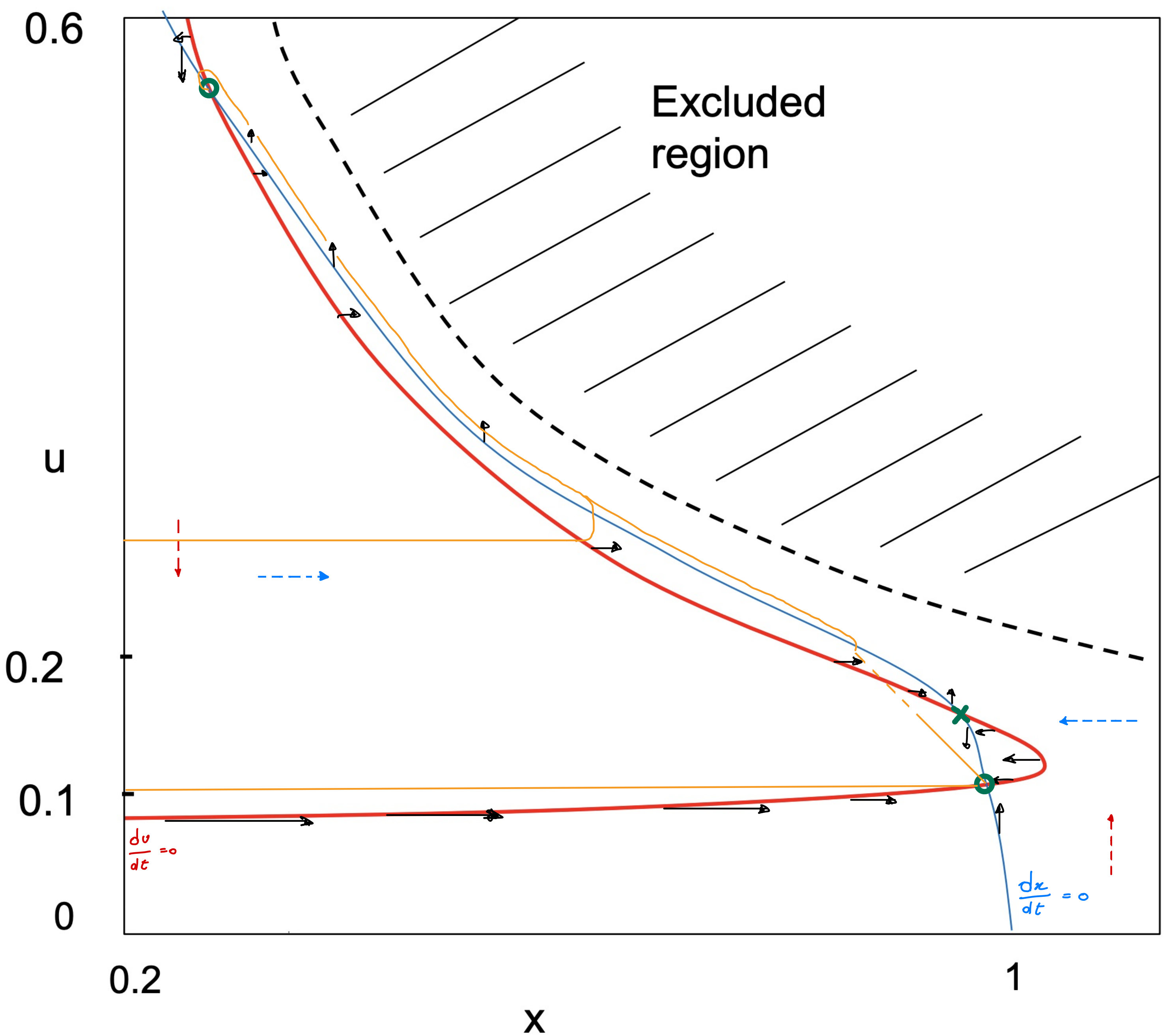
Exercise 1



d) In $(1, 0)$:

$$\frac{dx}{dt} = -\frac{0}{\cdots} - \cdots \frac{0}{\cdots} = 0$$

$$\frac{du}{dt} = -\frac{0.05}{0.7} + 0.05 \cdot 0.85 \frac{1}{1-0} =$$

$$= 0.05\left(\frac{1}{0.7} + 0.85\right) \cong 0.05\,(1.4 + 0.8) =$$

$$= 0.05 \cdot 2.2 \cong 0.1$$

b) $\frac{dx}{dt} = 0$ $\qquad$ $\frac{dv}{dt} = 0$

c) In $(0.9, 0)$

$$\frac{dx}{dt} = -\frac{0.9 - 1}{0.1} + \cdots \frac{0}{\cdots} =$$

$$-\frac{-0.1}{0.1} = 1$$

d) See graph!

e) See graph!

f) See graph!

g) See graph!

Exercise 2

a) We have

$$t_h \frac{dh}{dt} = -h + Jxy R/A + (I_0/A) =$$

$$= -h + Jxy\, Ah/A + I_0/A =$$

$$= -h(1 - Jxy) + I_0/A$$

$$\frac{t_h}{1-Jxy} \frac{dh}{dt} = -h + \frac{I_0}{A(1-Jxy)}$$

So h quickly and exponentially converges:

$$h = \frac{I_0}{A(1-Jxy)}$$

From which follows

$$\frac{dy}{dt} = -\frac{y-B}{t_y} + B(1-y)\frac{I_0 \cancel{A}}{\cancel{A}(1-Jxy)}$$

$$\frac{dx}{dt} = -\frac{x-1}{t_x} - \frac{xy I_0 \cancel{A}}{\cancel{A}(1-Jxy)}$$

b) $x \to x \qquad t_x \to t_r$

$y \to v \qquad t_y \to t_\delta$

where is c...?!

d) We went

$$t_h \frac{dh}{dt} = -h + Jxy \overbrace{R}^{= A\max(h,0) = 0}/A + I_0/A = 0$$

$$h = I_0/A \quad \leq 0$$

$$\frac{dy}{dt} = -\frac{y-B}{t_y} + B(1-y)\overbrace{R}^{=0} = 0 \qquad y = B$$

$$\frac{dx}{dt} = -\frac{x-1}{t_x} - xy\overset{=0}{R} = 0 \longrightarrow x = 1$$

e) At the beginnig, we are at the fixed point:

$$h = I_0/A \ll 0$$

$$x = 1$$

$$y = B$$

→ First h goes up quickly, becomig positive

→ Second, x goes down at medium speed.

→ third, y goes up slowly.

x is an inhibiting variable

y is an activation variable ?

time constents are weird!

Exercise 3

$$I = I_{ext} + I_{syn}$$

a)

$$\rightarrow \quad I(t_0) = \sum_{n}^{N} \sum_{f}^{F_n} w_0 \Big[\exp\big(-(t_0 - t_f^n)/\tau_2\big) + \\ - \exp\big(-(t_0 - t_f^n)/\tau_1\big) \Big]$$

$\rightarrow$ We have

$$A(t_0) = \frac{1}{N} \sum_{n}^{N} \sum_{f}^{F_n} \delta(t_0 - t_f^n)$$

So

$$I(t_0) = N w_0 \int_{-\infty}^{t_0} \Big[\exp\big(-(t_0 - t)/\tau_2\big) + \\ - \exp\big(-(t_0 - t)/\tau_1\big) \Big] \, dt$$

b) A single spike deposits

$$q = \int_0^\infty \gamma(t)\, dt =$$

$$= w_0 \int_0^\infty e^{-t/\tau_1} - e^{-t/\tau_2}\, dt =$$

$$= w_0 (\tau_2 - \tau_1)$$

$[A] = Hz$

For a given activity A all neurons produce a current

$$I_{syn} = N A q = A J_0 (\tau_2 - \tau_1)$$

Plugging the values we have

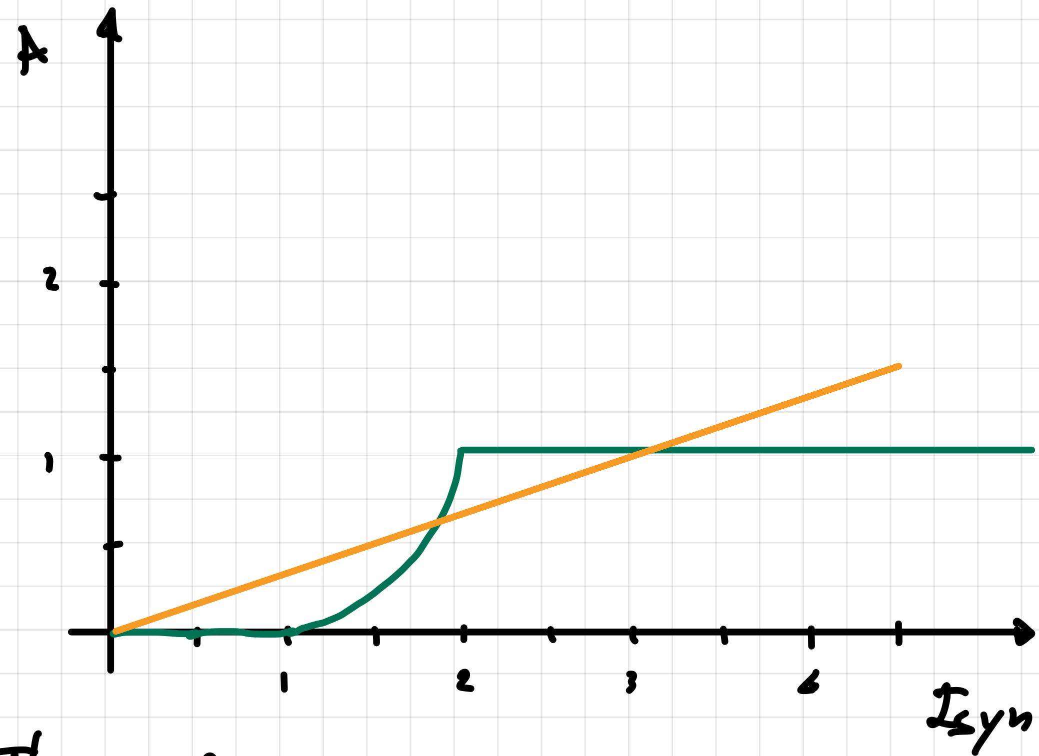
$$I_{syn} = 3A \longrightarrow A = \frac{1}{3} I_{syn} \; *$$

On the other hand we have

$$* \; A = g(I_{ext} + I_{syn})$$

So we can plot

$I_{ext} = 0$

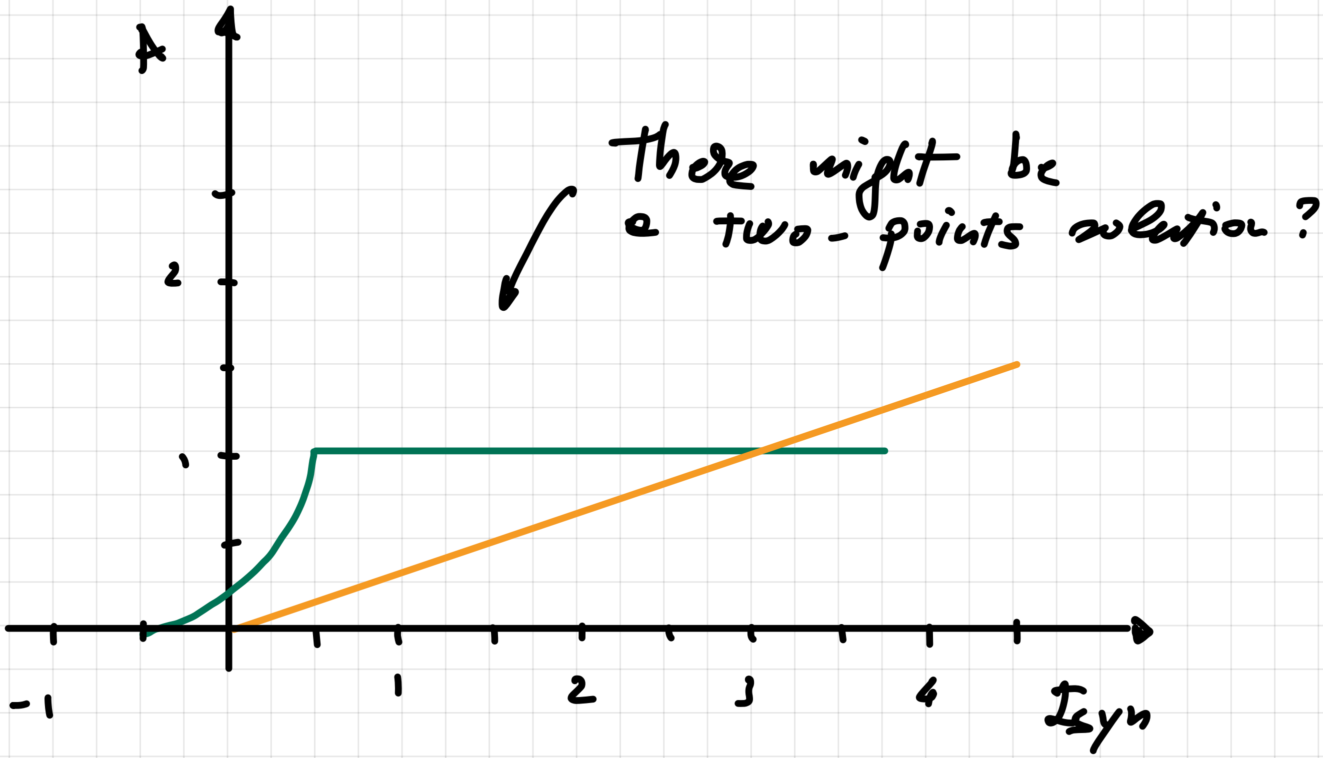


c) Three fixed points.

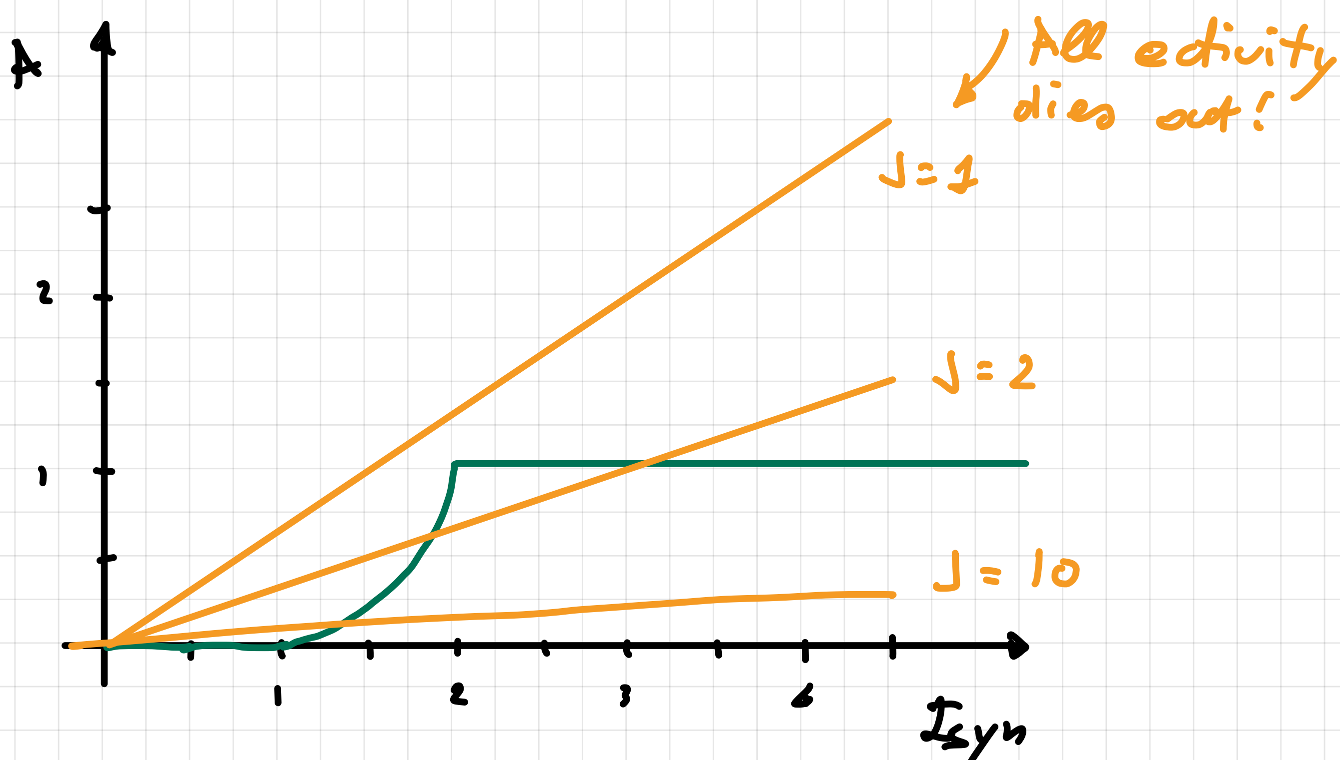
d) For $I_{ext} = 0.5$, $I_{syn} = I - 0.5$

Still three fixed points!

If we increase I_{ext} enough, however, we get only one fixed point!



d) If we set different values for J_0 we change the slope of the line:



Exercise 4

a) for $t > t_0$

$$\nu(t) = (v - E)^2 a \left(1 - e^{-\frac{(t - t_0)}{\tau}}\right)$$

b) With $T \gg \tau$, we have $\nu(t) = \nu_0(v)$ for most of the time.

Between closing and next opening:

$$f(t) = \frac{1}{\nu_0} e^{-\nu_0 t}$$

$\nu_0 = (v - E^2) a$

probability density

waiting time

Between openings:

$$f'(t) = \begin{cases} f(t - t_d) & \text{if } t > t_d \\ 0 & \text{otherwise} \end{cases}$$

opening to opening

c) Every round we have:

→ open for t_d time

→ closed for an amount of time distributed as

$$f(t) = t_d \, e^{-t/t_d}$$

Because the expected time spent closed is t_d, between two openings we spend t_d time open and t_d time closed: the channel is open 50% of the time!

time open

d)

$$m_o(v) = \frac{t_d}{t_d + \nu_o(v)^{-1}}$$

time open

(Expected) time closed

$v \to \infty \quad m_o \to 1$ (always open)

$v \to E \quad m_o \to 0$ (always closed)